

Structure of divergences in Drell-Yan process with small transverse momentum

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Abstract

We consider the structure of divergences in Drell-Yan process with small transverse momentum. The factorization proof is not trivial because various kinds of divergences are intertwined in the collinear and soft parts at high orders. We prescribe a method to disentangle the divergences in the framework of the soft-collinear effective theory. The rapidity divergence is handled by introducing the δ regulator in the collinear Wilson lines. The collinear part, which consists of the transverse-momentum-dependent parton distribution function (TMDPDF), is free of the rapidity divergence after the soft zero-bin subtraction. There still remains the problem of mixing between the ultraviolet and infrared divergences, which forbids the renormalization group description. We show that the mixing is cancelled by the soft function. This suggests that the collinear and soft parts should be treated as a whole in constructing a consistent factorization theorem. The renormalization group behavior of the combined collinear and soft parts is presented explicitly at one loop. We also show that the integrated PDF can be obtained by integrating the TMDPDF over the transverse momentum.

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I. INTRODUCTION

Theoretical predictions of high-energy scattering rely on the factorization of scattering cross sections, in which the hard, collinear and soft parts are separated to all orders in perturbation theory. Many inclusive cross sections have been proved to be factorized, and are used to compare with experiments. Less inclusive scattering processes are also of interest. Recently the transverse momentum distribution in the Higgs, Z boson, and $t\bar{t}$ pair production has redrawn new interest both theoretically [1–5] and experimentally [6, 7]. But in high-energy scattering processes with nonzero momentum transverse to the beam direction, the factorization proof becomes more involved. For example, in the Higgs production with the transverse momentum q_T with $\Lambda_{\text{QCD}} \ll q_T \ll M$, where M is the Higgs mass, the scattering cross section involves unintegrated, or transverse-momentum-dependent parton distribution functions (TMDPDFs). The study of the transverse momentum distribution was first heralded by Ref. [8]. However, the factorization proof, which appears ubiquitously in high-energy processes including Drell-Yan process, has been recently under debate again.

Before we discuss many issues in proving factorization of the processes with transverse momentum, it is worth delineating why the Drell-Yan process with small transverse momentum is so intricate. When we consider radiative corrections in massless gauge theories, there appear ultraviolet (UV) and infrared (IR) divergences. The UV and IR divergences should appear separately with no mixing in order for the theory to be consistent. In many inclusive processes, this was shown explicitly in the collinear and soft parts. In the process where the transverse momentum for a final-state particle is fixed, the UV divergence to be obtained by integrating over the transverse momentum does not emerge yet. Therefore the separation of the UV and IR divergences seems murky at first sight. Furthermore, when the transverse momentum is fixed, there appears another type of divergence called rapidity divergence, or lightcone singularity. The issues in the Drell-Yan processes with small transverse momentum reside in the appropriate treatment of rapidity divergence and in finding an adequate formalism in which the separation of the UV and IR divergences is established.

The first issue is the existence of rapidity divergence, which has been known for a long time [9], and various ways of handling it have been suggested. The source of the rapidity divergence can be understood by the following argument: In QCD, the Kinoshita-Lee-Nauenberg theorem [10, 11] guarantees that the divergence in virtual corrections at a given order in the strong coupling α_s is cancelled by the divergence in real gluon emissions, thus rendering inclusive scattering cross sections free of infrared divergence. The presence of infrared divergences can be traced by considering the kinematics in each case. In virtual corrections, the loop momentum can be collinear to an energetic particle, or soft, which results in collinear or soft divergences, or it can be both collinear and soft. In real emissions, the emitted gluon can also be collinear to an energetic particle or soft, or both as long as the phase space allows it. These divergences cancel in inclusive processes, and the inclusive scattering cross sections are physically meaningful in the absence of the collinear and soft divergences. However, if we consider differential cross sections with fixed transverse momentum, this cancellation becomes incomplete. That is, when the transverse momentum of an emitted gluon is fixed, the phase space of the emitted gluon does not cover all the available phase space in virtual

corrections, and the IR divergences do not cancel because of the imbalance of the phase space between real and virtual corrections. The presence of the infrared divergence due to this incomplete cancellation is referred to as the “rapidity divergence” or lightcone singularity since it is the result caused by the interaction of the energetic particle with collinear gluons with infinite rapidity. The dimensional regularization cannot regulate the rapidity divergence.

The difficulty in regulating the rapidity divergence was first noticed by Collins [9]. Collins and Soper [12, 13] suggested an intriguing idea of defining the operator whose matrix elements generate TMDPDF by tilting the Wilson lines which connect the fermion fields to make the operator gauge invariant. In association with this tilting, they introduced an additional scale of which the TMDPDF should be independent, and the corresponding evolution equation was developed. This method, while effective in handling the rapidity divergence, has a nuisance. First, the dependence of the additional scale appears in the hard, collinear and soft parts, which troubles the factorization proof. Though the dependence of the additional scale disappears when all the parts are summed, it is not clear how to approach the physical limit, that is, the limit in which all the particles are put on the lightcone. Finally it is not straightforward to obtain the integrated PDF from the TMDPDF by integrating over the transverse momentum [14].

Chiu et al. [15, 16] developed the idea of rapidity renormalization to handle the rapidity divergence. In high-energy scattering, there are collinear particles and soft particles, and they note that particles with the same offshellness can be labeled as collinear or soft depending on their rapidities. Therefore a rapidity scale is set up to distinguish collinear and soft particles. However, since the physics is independent of this arbitrary scale for the separation, the evolution equations of the collinear and the soft parts can be obtained with respect to the scaling of this rapidity scale. Technically they modified collinear and soft Wilson lines in such a way that Wilson lines depend on the rapidity scale. The rapidity scale dependence of the Wilson line is determined to satisfy the requirement that the cross section be independent of the rapidity scale, that is, the rapidity scale dependence of the collinear part is cancelled by that of the soft part. Though the motivations come from different reasoning, the approaches of Collins et al. [8] and Chiu et al. [15, 16] share in common the fact that they introduced an additional scale of which physical results should be independent. This is reflected in the evolution equations of the collinear and soft parts. The purpose of this approach is also focused to handle the rapidity divergence.

In this paper, a straightforward way of regulating the rapidity divergence is presented in the framework of the soft-collinear effective theory (SCET) [17–19]. The regularization for the rapidity divergence is achieved by introducing the δ regulator in the collinear Wilson lines only and not in any other propagators. Chiu et al. [20] introduced the δ regulator, which is an infrared regulator inserted in every propagator, as well as in the Wilson lines. They showed that the δ dependence of the collinear part in the back-to-back current is cancelled by the zero-bin subtraction. They stressed the importance of the proper zero-bin subtraction in the cancellation of the δ dependence. The δ regulators can be given as arbitrary parameters, or they can be related to the offshellness of the external particles. In either case, the δ dependence in the collinear and soft parts is different and cannot be

cancelled unless a specific relation is imposed. A simple relation can be found in a special case like a back-to-back current when the offshellness is employed to regulate divergences. However, when multijets are involved, δ depends on the offshellness of all the external particles in a complicated way. The collinear Wilson line in the n direction is obtained by considering the emission of n -collinear gluons from the other particles not in the n direction. Then the intermediate states become offshell, and the collinear Wilson line is obtained by integrating out this offshellness, and taking the leading term. If a back-to-back current is involved only, the δ regulator can be related to the offshellness of the particle in the \bar{n} direction for the n -collinear Wilson line. If we consider a process in which there are many jets, the offshellness in a collinear Wilson line depends on all the other jets in the process. On the other hand, the n -soft Wilson line is obtained by considering soft gluon emissions from the n -collinear particle. Therefore if the offshellness is included, the soft Wilson line involves only the offshellness of the corresponding collinear particle. Here the rapidity divergence is handled by inserting the δ regulator in the collinear Wilson lines only. Other regulators, though similar to the δ regulator in the collinear Wilson lines in form, regulate IR divergences.

The second issue is to separate the remaining UV and IR divergences. The presence of the mixing between the UV and IR divergences is troublesome because the physics at a high scale and a low scale is mixed, which does not make sense. Therefore the decoupling of the UV and IR divergences in loop calculations is essential to guarantee the consistency of the theory. In contrast to inclusive processes, the appropriate combination in which this decoupling occurs is still vague. It makes the definition of the TMDPDF itself controversial. The problem is whether the soft contribution or part of it should be included in the matrix element of the collinear operators which comprise the backbone of the TMDPDF. The underlying idea is to find a suitable combination of collinear and soft parts such that UV and IR divergences at higher orders are separated in order to prove factorization. A specific combination of collinear and soft parts has been claimed to achieve this separation [21]. But once soft part is included in the TMDPDF, there follows another issue, the universality of the TMDPDF. The soft part interacts with all the different collinear sectors, and it is different for different final hadronic states. Therefore the definition of the TMDPDF in one process may lose its meaning in other processes once the soft part in a specific process is included in the TMDPDF though the TMDPDFs in two different processes can be related to each other. This kind of relation for the PDF in deep inelastic scattering and Drell-Yan processes has been discussed when the soft part is included in the PDF [22].

In Ref. [23], an intriguing method to define the factorized TMDPDF was developed. First, the authors put all the particles on the lightcone, which is physical. Therefore there is no need to introduce additional scales due to the tilting of Wilson lines or rapidity separation. Obviously a technique to handle the rapidity divergence is necessary, and the δ regulator, slightly different from that in Ref. [20], is employed. They defined the TMDPDF in terms of the collinear part with the soft function, which is similar to the approach of Collins [21]. The reason is that the δ dependence, which indicates the presence of the rapidity divergence, is cancelled only through a specific combination of the collinear and soft parts as they claimed. It is based on the observation [24–26] that the zero-bin contribution can be identified as the

soft part. It is true for back-to-back currents to set up a relation of the regulators in the collinear and soft parts, but in general it does not hold. For example, if there are many jets or heavy colored particles in the process, there is no simple relation between them.

In this paper, the TMDPDF is defined in terms of the collinear fields only, thus it is universal in all high-energy processes. All the particles are put on the lightcone and the rapidity divergence is handled by the δ regulator in the collinear Wilson lines. The collinear part itself along with the corresponding zero-bin subtraction is independent of the δ regulator. The zero-bin contribution is performed in the limit where collinear particles become soft (not ultrasoft) to avoid double counting [27]. The infrared divergence from the soft Wilson lines is cancelled by that of the collinear part, which is controlled by the offshellness of the external particles. Combining all the ingredients, we obtain a factorized form of the scattering cross section with small transverse momentum in Drell-Yan processes. The hard part is the Wilson coefficients obtained in matching the current operators between QCD and SCET. The collinear part consists of the TMDPDF, and the remaining part is the soft function. We develop a method to handle the rapidity divergence, with the decoupling of the UV and IR divergences, and show the result explicitly at next-to-leading order (NLO) accuracy.

The structure of the paper is as follows: In Section II, the factorized form of the scattering cross section in Drell-Yan process is presented. In Section III, the TMDPDF is defined as the matrix elements of collinear fields, and the one-loop corrections are computed. The role of the zero-bin subtraction to avoid double counting is discussed and we argue that the soft zero-bin subtraction is the appropriate method for small transverse momentum $q_T \sim Q\lambda$. In Section IV, the soft function is defined in terms of the soft Wilson lines, and its one-loop correction is presented. The relation between the TMDPDF and the integrated PDF is discussed in Section V, and we explain how the integrated PDF is obtained from the TMDPDF. In Section VI, the renormalization group behavior of the scattering cross section at NLO is presented. In Section VII, the decoupling of the UV and IR divergences is elucidated in detail. And finally in Section VIII, we summarize the procedure for taming the rapidity divergence with the decoupling of the UV and IR divergences, and give a conclusion. In Appendix, the properties of the μ^2 distribution and the infinity distribution are described.

II. FACTORIZATION IN DRELL-YAN PROCESS

Let us consider Drell-Yan process $p\bar{p} \rightarrow \ell^+\ell^- + X$, where the lepton pair $\ell^+\ell^-$ is produced with small transverse momentum q_T ($\Lambda_{\text{QCD}} \ll q_T \ll Q$). The invariant mass of the lepton pair Q is the large scale. The incoming particles are n -collinear and \bar{n} -collinear, and their momenta scale as

$$p_n^\mu = (\bar{n} \cdot p_n, n \cdot p_n, p_{n\perp}) \sim Q(1, \lambda^2, \lambda), \quad p_{\bar{n}}^\mu = (\bar{n} \cdot p_{\bar{n}}, n \cdot p_{\bar{n}}, p_{\bar{n}\perp}) \sim Q(\lambda^2, 1, \lambda), \quad (1)$$

where $\lambda = q_T/Q$ is the small parameter. We also denote $\bar{n} \cdot p = p^-$, $n \cdot p = p^+$. There are soft particles, whose momenta scale as $p_s^\mu \sim Q(\lambda, \lambda, \lambda)$, in the final state to balance the transverse momentum of the lepton pair. In SCET, the interactions between collinear and

soft particles are not allowed since they put each particle far off their mass shells. Therefore the soft part is decoupled and is expressed in terms of the soft Wilson lines. The ultrasoft (usoft) particles can also exist, but they do not contribute to the cross section with transverse momentum $q_T \sim Q\lambda$.

The differential scattering cross section can be written as

$$d\sigma = \frac{\sigma_0(q^2)}{s} \frac{d^4q}{(2\pi)^4} \int d^4x e^{-iq \cdot x} (-g^{\mu\nu} N_c) \langle N_1 N_2 | J_\mu^\dagger(x) J_\nu(0) | N_1 N_2 \rangle, \quad (2)$$

where $\sigma_0 = 4\pi\alpha^2 Q_f^2 / (3q^2 N_c)$, with the electric charge Q_f and the number of colors N_c . The electromagnetic current is given by $J_\mu = \bar{f} \gamma^\mu f$. The sum over the flavors f is implied. We consider only the electroproduction here, but the weak interaction can be easily implemented. We choose the direction of the momentum P_1 for the nucleon N_1 to be in the n direction, and that of P_2 for N_2 in the \bar{n} direction. In SCET, the current operator for the quark-antiquark annihilation can be written as

$$J_\mu(x) = C(Q) e^{-i(\bar{n} \cdot p n \cdot x / 2 + n \cdot \bar{p} \bar{n} \cdot x / 2)} \bar{\chi}_{n,p} Y_n^\dagger \gamma_\mu Y_{\bar{n}} \chi_{\bar{n},\bar{p}}(\mathbf{x}_\perp), \quad (3)$$

where $Q^2 = q^2$, and $\chi_n = W_n^\dagger \xi_n$ is the gauge-invariant collinear fermion field with the collinear Wilson line W_n . $C(Q)$ is the Wilson coefficient obtained in matching between the full theory and SCET. Unlike the conventional SCET formalism in which the momenta of order Q and $Q\lambda$ are label momenta, only the momenta of order Q are the only label momenta to be extracted in Eq. (3). It is because the soft momentum of order $Q\lambda$ is the dynamical degree of freedom, and the size of the fluctuation after extracting label momenta is of order $\mathbf{x}_\perp \sim (Q\lambda)^{-1}$.

The differential cross section in SCET can be written as

$$\begin{aligned} d\sigma &= \frac{d^4q}{(2\pi)^4} \frac{\sigma_0}{s} (-g^{\mu\nu} N_c) \int d\omega_1 d\omega_2 \int d^4x e^{-iq \cdot x} e^{i(\omega_1 n \cdot x / 2 + \omega_2 \bar{n} \cdot x / 2)} |C(Q)|^2 \\ &\times \langle N_1(P_1) N_2(P_2) | [\bar{\chi}_n \delta(\omega_1 - \bar{n} \cdot \mathcal{P}^\dagger)] Y_n^\dagger \gamma_{\perp\mu} Y_{\bar{n}} [\delta(\omega_2 + n \cdot \mathcal{P}) \chi_{\bar{n}}(\mathbf{x}_\perp)] \\ &\times \bar{\chi}_{\bar{n}} Y_{\bar{n}}^\dagger \gamma_{\perp\nu} Y_n \chi_n(0) | N_1(P_1) N_2(P_2) \rangle, \end{aligned} \quad (4)$$

where $\omega_1 = x_1 \bar{n} \cdot P_1$, $\omega_2 = x_2 n \cdot P_2$, and x_1, x_2 are the longitudinal momentum fractions of the incoming partons. Here $\bar{n} \cdot \mathcal{P}$, $n \cdot \mathcal{P}$ are the label momentum operators for the particles in the n and \bar{n} directions respectively and the operators are applied inside the brackets. Since the soft Wilson lines are decoupled from collinear particles, they can be extracted out, and expressed in terms of the vacuum expectation values. The fields at \mathbf{x}_\perp can be expressed by the fields at the origin as

$$\begin{aligned} (\bar{\chi}_n)_\alpha^a(\mathbf{x}_\perp) &= \int d^2\mathbf{k}_{1\perp} e^{-i\mathbf{k}_{1\perp} \cdot \mathbf{x}_\perp} [(\bar{\chi}_n)_\alpha^a(0) \delta^{(2)}(\mathbf{k}_{1\perp} - \mathcal{P}_\perp^\dagger)], \\ (\chi_{\bar{n}})_\alpha^a(\mathbf{x}_\perp) &= \int d^2\mathbf{k}_{2\perp} e^{-i\mathbf{k}_{2\perp} \cdot \mathbf{x}_\perp} [\delta^{(2)}(\mathbf{k}_{2\perp} + \mathcal{P}_\perp) (\chi_{\bar{n}})_\alpha^a(0)], \\ (Y_n^\dagger Y_{\bar{n}})_{ab}(\mathbf{x}_\perp) (Y_{\bar{n}} Y_n)_{cd}(0) &= \int d^2\boldsymbol{\eta}_\perp e^{i\boldsymbol{\eta}_\perp \cdot \mathbf{x}_\perp} (Y_n^\dagger Y_{\bar{n}})_{ab} \delta^{(2)}(\boldsymbol{\eta}_\perp + i\nabla_\perp) (Y_{\bar{n}}^\dagger Y_n)_{cd}(0), \end{aligned} \quad (5)$$

where \mathcal{P}_\perp is the operator extracting the transverse momentum. The TMDPDFs are defined after taking the spin average as

$$\begin{aligned}\langle N_1 | (\chi_n)_\alpha^a \left[(\bar{\chi}_n)_\beta^b \delta \left(x_1 - \frac{\bar{n} \cdot \mathcal{P}^\dagger}{\bar{n} \cdot P_1} \right) \delta^{(2)}(\mathbf{k}_{1\perp} - \mathcal{P}_\perp^\dagger) \right] | N_1 \rangle &= \frac{\bar{n} \cdot P_1}{2N_c} \delta^{ab} \left(\frac{\not{n}}{2} \right)_{\alpha\beta} f_{q/N_1}(x_1, \mathbf{k}_{1\perp}), \\ \langle N_2 | \left[\delta \left(x_2 + \frac{n \cdot \mathcal{P}}{n \cdot P_2} \right) \delta^{(2)}(\mathbf{k}_{2\perp} + \mathcal{P}_\perp) (\chi_{\bar{n}})_\alpha^a \right] (\bar{\chi}_{\bar{n}})_\beta^b | N_2 \rangle &= \frac{n \cdot P_2}{2N_c} \delta^{ab} \left(\frac{\not{n}}{2} \right)_{\alpha\beta} f_{\bar{q}/N_2}(x_2, \mathbf{k}_{2\perp}).\end{aligned}\quad (6)$$

After straightforward algebra, the differential scattering cross section can be written as

$$\begin{aligned}\frac{d\sigma}{d^2\mathbf{q}_\perp} &= \int dx_1 dx_2 \sigma_0 H(Q^2, \mu) \int d^2\mathbf{k}_{1\perp} d^2\mathbf{k}_{2\perp} d^2\boldsymbol{\eta}_\perp \delta^{(2)}(\mathbf{k}_{1\perp} + \mathbf{k}_{2\perp} - \boldsymbol{\eta}_\perp - \mathbf{q}_\perp) \\ &\quad \times f_{q/N_1}(x_1, \mathbf{k}_{1\perp}) f_{\bar{q}/N_2}(x_2, \mathbf{k}_{2\perp}) S(\boldsymbol{\eta}_\perp),\end{aligned}\quad (7)$$

where $s = \bar{n} \cdot P_1 n \cdot P_2$, $Q^2 = x_1 x_2 s$, and \mathbf{q}_\perp is the transverse momentum of the lepton pair. From Eq. (6), the TMDPDFs are given as

$$\begin{aligned}f_{q/N_1}(x, \mathbf{k}_\perp) &= \langle N_1(P_1) | \bar{\chi}_n \frac{\not{n}}{2} \delta^{(2)}(\mathbf{k}_\perp - \mathcal{P}_\perp) \delta(\bar{n} \cdot \mathcal{P} - x \bar{n} \cdot P_1) \chi_n | N_1(P_1) \rangle, \\ f_{\bar{q}/N_2}(x, \mathbf{k}_\perp) &= \langle N_2(P_2) | \bar{\chi}_{\bar{n}} \frac{\not{n}}{2} \delta^{(2)}(\mathbf{k}_\perp + \mathcal{P}_\perp) \delta(n \cdot \mathcal{P} + x n \cdot P_2) \chi_{\bar{n}} | N_2(P_2) \rangle,\end{aligned}\quad (8)$$

and the soft Wilson line is given by

$$S(\boldsymbol{\eta}_\perp) = \frac{1}{N_c} \langle 0 | \text{tr} \left[Y_n^\dagger Y_{\bar{n}} \delta^{(2)}(\boldsymbol{\eta}_\perp + i \nabla_\perp) Y_{\bar{n}}^\dagger Y_n \right] | 0 \rangle. \quad (9)$$

There is an alternative definition of TMDPDF in previous literature in terms of the delta functions of the transverse momentum in $D - 2$ dimensions instead of two dimensions. With this definition, the radiative corrections should change accordingly, but equivalent results can be obtained. However, the definition in Eq. (8) is preferred in this paper since the TMDPDF is an observable and it should be defined in four spacetime dimensions. If the Wilson lines Y_n and $Y_{\bar{n}}$ are usoft, the momentum fluctuation is of order $Q\lambda^2$ and the momentum operator in the delta function in Eq. (9) can be put to zero at leading order, and the usoft Wilson lines cancel. The fact that usoft interactions do not contribute to the process with the transverse momentum of order $Q\lambda$ to all orders in α_s manifests itself due to the cancellation of the usoft Wilson lines.

Eq. (7) is the factorized form for the differential scattering cross section. The hard function $H(Q) = |C(Q)|^2$ is the Wilson coefficient of the current, the collinear part consists of the product of two TMDPDFs, and the soft Wilson lines comprise the soft part. The radiative corrections of the hard, collinear and soft parts can be computed separately in perturbation theory. However, the infrared divergences, or the dependence of the regulators in actual calculations are intertwined between the collinear and soft parts, of which the disentanglement is the main topic of this paper. It will be shown that the dependence of the δ regulator, which controls the rapidity divergence, cancels in the collinear sector. The mixing between the UV and IR divergences disappears when the virtual and real corrections are added in the collinear sector and in the soft sector. It also disappears when the collinear

and soft parts are added in the virtual corrections and in the real gluon emissions. This observation implies the separation of divergences to all orders.

The TMDPDF can be expressed in terms of the integrated PDF $\phi_{q/N}(z)$ as

$$f_{q/N}(x, \mathbf{k}_\perp^2, \mu) = \int_x^1 \frac{dz}{z} J_{qq}(z, \mathbf{k}_\perp^2, \mu) \phi_{q/N}\left(\frac{x}{z}, \mu\right) = \int_x^1 \frac{dz}{z} J_{qq}\left(\frac{x}{z}, \mathbf{k}_\perp^2, \mu\right) \phi_{q/N}(z, \mu). \quad (10)$$

This corresponds to the matching of the operator for the TMDPDF at the scale $\sim Q\lambda$ to the operator for the integrated PDF at the scale $\sim Q\lambda^2$, and the kernel $J_{qq}(z, \mathbf{k}_\perp^2, \mu)$ is the matching coefficient. The differential cross section in Eq. (7) can be expressed in terms of the integrated PDF as

$$\begin{aligned} \frac{d\sigma}{d^2\mathbf{q}_\perp} &= \int dx_1 dx_2 \sigma_0 H(Q^2, \mu) \int d^2\mathbf{k}_{1\perp} d^2\mathbf{k}_{2\perp} d^2\boldsymbol{\eta}_\perp \delta^{(2)}(\mathbf{k}_{1\perp} + \mathbf{k}_{2\perp} - \boldsymbol{\eta}_\perp - \mathbf{q}_\perp) S(\boldsymbol{\eta}_\perp, \mu) \quad (11) \\ &\times \int_{x_1}^1 \frac{dz_1}{z_1} \int_{x_2}^1 \frac{dz_2}{z_2} J_{qq}(z_1, \mathbf{k}_{1\perp}, \mu) J_{\bar{q}\bar{q}}(z_2, \mathbf{k}_{2\perp}, \mu) \phi_{q/N_1}\left(\frac{x_1}{z_1}, \mu\right) \phi_{\bar{q}/N_2}\left(\frac{x_2}{z_2}, \mu\right) \\ &= \int dx_1 dx_2 \sigma_0 H(Q^2, \mu) \int_{x_1}^1 \frac{dz_1}{z_1} \int_{x_2}^1 \frac{dz_2}{z_2} W(z_1, z_2, \mathbf{q}_\perp, \mu) \phi_{q/N_1}\left(\frac{x_1}{z_1}, \mu\right) \phi_{\bar{q}/N_2}\left(\frac{x_2}{z_2}, \mu\right), \end{aligned}$$

where the TMD kernel $W(z_1, z_2, \mathbf{q}_\perp, \mu)$ is defined as

$$\begin{aligned} W(z_1, z_2, \mathbf{q}_\perp, \mu) &= \int d^2\mathbf{k}_{1\perp} d^2\mathbf{k}_{2\perp} d^2\boldsymbol{\eta}_\perp \delta^{(2)}(\mathbf{k}_{1\perp} + \mathbf{k}_{2\perp} - \boldsymbol{\eta}_\perp - \mathbf{q}_\perp) \\ &\times S(\boldsymbol{\eta}_\perp, \mu) J_{qq}(z_1, \mathbf{k}_{1\perp}, \mu) J_{\bar{q}\bar{q}}(z_2, \mathbf{k}_{2\perp}, \mu). \quad (12) \end{aligned}$$

All the information on the transverse momentum resides in $W(z_1, z_2, \mathbf{q}_\perp)$, which should be IR finite. It will be computed at next-to-leading order.

The anomalous dimension for W at one loop is given as

$$\begin{aligned} \gamma_W &= \mu \frac{d}{d\mu} W(z_1, z_2, \mathbf{q}_\perp, \mu) \\ &= \gamma_S(\mathbf{q}_\perp^2) \delta(1 - z_1) \delta(1 - z_2) + \gamma_{J_{qq}}(z_1, \mathbf{q}_\perp^2) \delta(1 - z_2) + \gamma_{J_{\bar{q}\bar{q}}}(z_2, \mathbf{q}_\perp^2) \delta(1 - z_1), \quad (13) \end{aligned}$$

where

$$\gamma_S = \mu \frac{d}{d\mu} S(\mathbf{q}_\perp, \mu), \quad \gamma_{J_{qq}} = \mu \frac{d}{d\mu} J_{qq}(z_1, \mathbf{q}_\perp, \mu), \quad \gamma_{J_{\bar{q}\bar{q}}} = \mu \frac{d}{d\mu} J_{\bar{q}\bar{q}}(z_2, \mathbf{q}_\perp, \mu). \quad (14)$$

The anomalous dimensions γ_S , $\gamma_{J_{qq}}$ and $\gamma_{J_{\bar{q}\bar{q}}}$ contain IR divergences, hence not physically meaningful. But the sum is independent of the IR divergence.

Summarizing our approach, all the particles are on their lightcones, and we define a universal TMDPDF in terms of collinear fields only, and employ the soft zero-bin subtraction in the TMDPDF to avoid double counting. It is different from other approaches in which soft parts are involved in the definition of the TMDPDF. If soft parts are included in the TMDPDF, its definition cannot be universal since soft parts are different in different scattering processes in general.

III. RADIATIVE CORRECTIONS OF THE TMDPDF

In SCET, the TMDPDF at the parton level is defined as

$$f_{q/N}(\frac{\omega}{p_-}, \mathbf{k}_\perp) = \langle N(p^-) | \bar{\chi}_n \frac{\not{n}}{2} \delta(\omega - \bar{n} \cdot \mathcal{P}) \delta^{(2)}(\mathbf{k}_\perp - \mathcal{P}_\perp) \chi_n | N(p^-) \rangle. \quad (15)$$

It is implied that $\omega \geq 0$, and the longitudinal momentum fraction x is given by $x = \omega/p^-$. The TMDPDF $f_{\bar{q}/N}$ can be treated in a similar way. As emphasized, it consists of the collinear fields only, which enables a universal definition of the TMDPDF. There is rapidity divergence, and it is regulated by the δ regulator in the collinear Wilson lines. After the zero-bin subtraction, the radiative correction is free of rapidity divergence as in the full theory.

The integrated PDF can be obtained by integrating the TMDPDF over the transverse momentum to all orders in α_s as

$$\phi_{q/N}(x) = \int d^2\mathbf{k}_\perp f_{q/N}(\frac{\omega}{p_-}, \mathbf{k}_\perp) = \langle N(p^-) | \bar{\chi}_n \frac{\not{n}}{2} \delta(\omega - \bar{n} \cdot \mathcal{P}) \chi_n | N(p^-) \rangle. \quad (16)$$

This is obviously true at tree level since the tree-level PDFs are given by

$$f_{q/N}^{(0)}(x, \mathbf{k}_\perp^2) = \delta(1-x) \delta^{(2)}(\mathbf{k}_\perp), \quad \phi_{q/N}^{(0)}(x) = \delta(1-x). \quad (17)$$

But one of the main issues involved in TMDPDF is whether it holds true at higher orders [14]. We are going to construct a formalism in which Eq. (16) works to all orders, and show the explicit result at one loop.

The δ regulator is introduced in the collinear Wilson lines W_n and $W_{\bar{n}}$ as

$$W_n = \sum_{\text{perm}} \exp\left[-\frac{g}{\bar{n} \cdot \mathcal{P} + \delta_1} \bar{n} \cdot A_n\right], \quad W_{\bar{n}} = \sum_{\text{perm}} \exp\left[-\frac{g}{n \cdot \mathcal{P} + \delta_2} n \cdot A_{\bar{n}}\right]. \quad (18)$$

The δ regulators, as discussed, are employed to regulate the rapidity divergence in TMDPDF. These regulators can be either arbitrary parameters independent of kinematics, or the quantities depending on the details of the jets other than the n or \bar{n} directions. In the special case for a back-to-back current, if we put n and \bar{n} -collinear particles slightly off shell by p_1^2 and p_2^2 , they are given by $\delta_1 = p_2^2/n \cdot p_2$ and $\delta_2 = p_1^2/\bar{n} \cdot p_1$ in the latter case.

The Feynman diagrams for the radiative corrections at one loop are shown in Fig. 1. Fig. 1 (a) gives

$$M_a = \frac{\alpha_s C_F}{2\pi^2} \delta(1-x) \delta(\mathbf{k}_\perp^2) \left[\frac{1}{\epsilon} \left(1 + \ln \frac{\delta_1}{p^-}\right) + \ln \frac{\mu^2}{-p^2} - \frac{1}{2} \ln^2 \frac{\delta_1}{p^-} + \ln \frac{\mu^2}{-p^2} \ln \frac{\delta_1}{p^-} + 2 - \frac{\pi^2}{3} \right]. \quad (19)$$

Here we use the relation $\delta^{(2)}(\mathbf{k}_\perp) = \delta(\mathbf{k}_\perp^2)/\pi$ since the remaining function is independent of the azimuthal angle. Fig. 1 (b) and (c) yield

$$M_b = \frac{\alpha_s C_F}{2\pi^2} \left\{ \delta(\mathbf{k}_\perp^2) \left[\left(\frac{x}{(1-x)_+} - \delta(1-x) \ln \frac{\delta_1}{p^-} \right) \ln \frac{\mu^2}{-p^2} \right. \right. \\ \left. \left. + \left(\frac{\pi^2}{6} + \frac{1}{2} \ln^2 \frac{\delta_1}{p^-} \right) \delta(1-x) - x \left(\frac{\ln(1-x)}{(1-x)_+} \right) - \frac{x \ln x}{(1-x)_+} \right] \right.$$

$$\begin{aligned}
& + \left[\frac{1}{\mathbf{k}_\perp^2} \right]_{\mu^2} \left(\frac{x}{(1-x)_+} - \delta(1-x) \ln \frac{\delta_1}{p^-} \right) \Big\}, \\
M_c = & \frac{\alpha_s C_F}{2\pi^2} (1-x) \left\{ \delta(\mathbf{k}_\perp^2) \left[\ln \frac{\mu^2}{-p^2} - 1 - \ln x(1-x) \right] + \left[\frac{1}{\mathbf{k}_\perp^2} \right]_{\mu^2} \right\}.
\end{aligned} \tag{20}$$

In Eq. (20), the μ^2 distribution function is introduced. For a well-behaved test function $f(\mathbf{k}_\perp^2)$, the integration involving the μ^2 distribution is defined as

$$\int_0^{\Lambda_T^2} d\mathbf{k}_\perp^2 [g(\mathbf{k}_\perp^2)]_{\mu^2} f(\mathbf{k}_\perp^2) = \int_0^{\Lambda_T^2} d\mathbf{k}_\perp^2 g(\mathbf{k}_\perp^2) f(\mathbf{k}_\perp^2) - \int_0^{\mu^2} d\mathbf{k}_\perp^2 g(\mathbf{k}_\perp^2) f(0), \tag{21}$$

where Λ_T is a finite upper limit for the relevant physical processes in consideration. Here $g(\mathbf{k}_\perp^2)$ is a function which diverges at $\mathbf{k}_\perp^2 = 0$. At one loop, there are two types of $g(\mathbf{k}_\perp^2)$ given by

$$g_1(\mathbf{k}_\perp^2) = \frac{1}{\mathbf{k}_\perp^2}, \quad g_2(\mathbf{k}_\perp^2) = \frac{\ln(\mathbf{k}_\perp^2/\mu^2)}{\mathbf{k}_\perp^2}. \tag{22}$$

Of course, any definition of the distribution function will do with an arbitrary upper limit in the second integral in Eq. (21) instead of μ^2 as long as it removes the IR singularity at $\mathbf{k}_\perp^2 = 0$. The choice of the μ^2 distribution is somewhat cosmetic in the sense that the μ dependence of the finite part is the same as the radiative corrections of the integrated PDF. And there appears no explicit dependence on Λ_T .

Since we require that $\mathbf{k}_\perp \sim \mathcal{O}(Q\lambda)$, the transverse momentum should remain finite. It is convenient to obtain the information about the dependence of the TMDPDF on the small transverse momentum from experiments. But in the effective theory, it can reach infinity and the UV divergence can be extracted. However, it becomes subtle if we allow the transverse momentum to have arbitrary values extending to infinity. In this case the distribution should have a different form and the UV divergence should be included. This is relevant when we try to obtain the integrated PDF from the TMDPDF by integrating the transverse momentum from 0 to infinity. Then the definition of the distribution function includes the UV divergence, and it is referred to as the “infinity distribution”. This will be discussed in the next section when we compare the radiative corrections of the TMDPDF

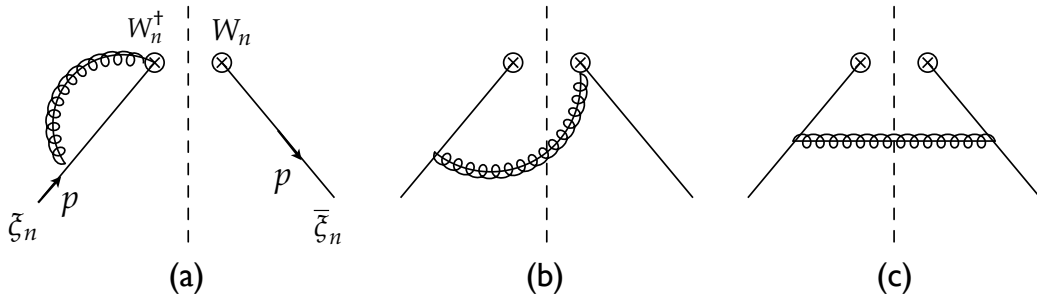


FIG. 1. Feynman diagrams for one-loop corrections of the TMDPDF (a) virtual corrections and (b), (c) real gluon emission. The mirror images of (a) and (b) should be included.

and the integrated PDF. The properties of these distribution functions are described in detail in Appendix A.

In computing collinear matrix elements, the kinematic region with soft momentum is also included. Since the soft contribution is computed separately, the contribution from the soft region to the collinear part should be subtracted to avoid double counting. This is referred to as the zero-bin subtraction [27]. However, there are two possible kinematic regions for the zero-bin contribution: soft particles with momentum $p_s \sim Q\lambda$, and usoft particles with momentum $p_{\text{us}} \sim Q\lambda^2$. The need for the distinction between the soft and usoft zero-bin contributions arises depending on whether the processes in consideration are sensitive to soft momentum or not. When the zero-bin subtraction was first applied to a heavy-to-light current [27], the soft interaction is decoupled from the current because the interaction puts a heavy or a collinear particle off the mass shell. The resultant effective theory is SCET_I. In order to go down to SCET_{II}, where the fields have fluctuations of order $Q\lambda^2$, all the soft scales of order $Q\lambda$ are rescaled to usoft scales of order $Q\lambda^2$. Therefore there is no need to distinguish soft and usoft zero-bin subtractions because there is a smooth transition from soft to usoft contributions as we scale down from $Q\lambda$ to $Q\lambda^2$.

When there is no smooth transition from soft to usoft region as in the Drell-Yan process with transverse momentum of order $Q\lambda$, care should be taken about which zero-bin subtraction should be performed. If we scale down from soft to usoft scales, while the transverse momentum is fixed $q_T \sim Q\lambda$, the resultant usoft interactions cannot produce transverse momentum of order $Q\lambda$. In other words, the soft part treats physics with momenta of order $Q\lambda$, and a naive computation of the collinear part includes the soft part which must belong to the soft sector. Therefore the soft zero-bin subtraction is appropriate to correctly avoid double counting in the collinear part. As we mentioned earlier, the usoft Wilson lines cancel and there is no usoft contribution.

The zero-bin contribution for M_a is given by

$$M_a^0 = \frac{\alpha_s C_F}{2\pi^2} \delta(1-x) \delta(\mathbf{k}_\perp^2) \left[-\frac{1}{\epsilon^2} - \frac{1}{\epsilon} \ln \frac{\mu^2 p^-}{-p^2 \delta_1} - \frac{1}{2} \ln^2 \frac{\mu^2 p^-}{-p^2 \delta_1} - \frac{\pi^2}{4} \right]. \quad (23)$$

The same result is obtained whether the soft or usoft zero-bin contribution is considered since there is no distinction in the virtual correction. The major difference comes from real gluon emission. The soft zero-bin contribution for M_b , responsible for real gluon emission, is given as

$$M_{b,\text{soft}}^0 = \frac{\alpha_s C_F}{2\pi^2} \delta(1-x) \left\{ \delta(\mathbf{k}_\perp^2) \left(\frac{1}{2} \ln^2 \frac{\mu^2 p^-}{-p^2 \delta_1} + \frac{\pi^2}{3} \right) + \left(\ln \frac{\mu^2}{-p^2} - \ln \frac{\delta_1}{p^-} \right) \left[\frac{1}{\mathbf{k}_\perp^2} \right]_{\mu^2} + \left[\frac{\ln \mathbf{k}_\perp^2 / \mu^2}{\mathbf{k}_\perp^2} \right]_{\mu^2} \right\}. \quad (24)$$

On the other hand, the corresponding usoft zero-bin contribution is given by

$$M_{b,\text{usoft}}^0 = \frac{\alpha_s C_F}{2\pi^2} \delta(1-x) \delta(\mathbf{k}_\perp^2) \left[\frac{1}{\epsilon^2} + \frac{1}{\epsilon} \ln \frac{\mu^2 p^-}{-p^2 \delta_1} + \frac{1}{2} \ln^2 \frac{\mu^2 p^-}{-p^2 \delta_1} + \frac{\pi^2}{4} \right]. \quad (25)$$

This is exactly the same as M_a^0 with the opposite sign. The zero-bin contribution for M_c is suppressed by λ compared to M_c and is neglected here. Combining Eqs. (19), (20), (23)

and (24), the collinear contributions after the soft zero-bin subtraction are given as

$$\begin{aligned}
\tilde{M}_a &= M_a - M_a^0 = \frac{\alpha_s C_F}{2\pi^2} \delta(1-x) \delta(\mathbf{k}_\perp^2) \left[\frac{1}{\epsilon^2} + \frac{1}{\epsilon} \left(1 + \ln \frac{\mu^2}{-p^2} \right) + \ln \frac{\mu^2}{-p^2} + \frac{1}{2} \ln^2 \frac{\mu^2}{-p^2} + 2 - \frac{\pi^2}{12} \right], \\
\tilde{M}_b &= M_b - M_{b,\text{soft}}^0 = \frac{\alpha_s C_F}{2\pi^2} \left\{ \delta(\mathbf{k}_\perp^2) \left[\delta(1-x) \left(-\frac{1}{2} \ln^2 \frac{\mu^2}{-p^2} - \frac{\pi^2}{6} \right) + \frac{x}{(1-x)_+} \left(\ln \frac{\mu^2}{-p^2} - \ln x \right) \right. \right. \\
&\quad \left. \left. - x \left(\frac{\ln(1-x)}{1-x} \right)_+ \right] + \left[\frac{1}{\mathbf{k}_\perp^2} \right]_{\mu^2} \left(\frac{x}{(1-x)_+} - \delta(1-x) \ln \frac{\mu^2}{-p^2} \right) - \delta(1-x) \left[\frac{1}{\mathbf{k}_\perp^2} \ln \frac{\mathbf{k}_\perp^2}{\mu^2} \right]_{\mu^2} \right\}, \\
\tilde{M}_c &= M_c = \frac{\alpha_s C_F}{2\pi^2} (1-x) \left\{ \delta(\mathbf{k}_\perp^2) \left[\ln \frac{\mu^2}{-p^2} - 1 - \ln x(1-x) \right] + \left[\frac{1}{\mathbf{k}_\perp^2} \right]_{\mu^2} \right\}. \tag{26}
\end{aligned}$$

Just for reference, \tilde{M}_b in the usoft zero-bin subtraction is given as

$$\begin{aligned}
\tilde{M}_{b,\text{usoft}} &= M_b - M_{b,\text{usoft}}^0 = \frac{\alpha_s C_F}{2\pi^2} \left\{ \delta(\mathbf{k}_\perp^2) \left[\delta(1-x) \left(-\frac{1}{2} \ln^2 \frac{\mu^2}{-p^2} - \frac{\pi^2}{12} \right) \right. \right. \\
&\quad \left. \left. + \frac{x}{(1-x)_+} \left(\ln \frac{\mu^2}{-p^2} - \ln x \right) - x \left(\frac{\ln(1-x)}{1-x} \right)_+ \right] \right. \\
&\quad \left. + \left[\frac{1}{\mathbf{k}_\perp^2} \right]_{\mu^2} \frac{x}{(1-x)_+} - \delta(1-x) \left[\frac{1}{\mathbf{k}_\perp^2} \right]_{\mu^2} \ln \frac{\delta_1}{p^-} \right\}. \tag{27}
\end{aligned}$$

Each Feynman diagram for the collinear contribution depends on the δ regulator, but after the soft zero-bin subtraction, the collinear contribution is independent of δ_1 . On the other hand, after the usoft zero-bin subtraction, there still remains the dependence on δ_1 . This reflects the fact that there is a mismatch in counting the degrees of freedom in the usoft zero-bin contribution and in the soft contribution. Furthermore, note that $M_a^0 + M_{b,\text{usoft}}^0 = 0$ from Eqs. (23) and (25), which confirms that there are no usoft contributions in this process.

To summarize, the soft zero-bin subtraction is appropriate not because it makes the result free of δ_1 , but because there is a definite physical reason. Since the collinear contribution with the soft zero-bin subtraction is free of the δ regulator, there is no need to relate the δ regulators in the collinear Wilson lines to the offshellness appearing in the soft Wilson lines. This is important because the treatment of the δ regulator can be applied to processes other than Drell-Yan process, in which there are collinear particles other than n and \bar{n} directions. The collinear Wilson line is obtained by integrating out the degrees of freedom of order Q when collinear gluons are emitted from other particles. For example, if there are distinct jets or heavy colored particles in the final state, the δ regulator for a specific collinear Wilson line depends, in general, on the offshellness of other jets. The leading eikonalized form is the same irrespective of the source of the emitted collinear gluons. If we try to incorporate the δ regulator by invoking the offshellness of other particles, the δ regulator depends in a complicated way on the offshellness of other particles. In our scheme, it is not necessary for the δ regulator in the collinear Wilson line to have a definite relationship with the offshellness in the soft part. The dependence on the δ regulator in the collinear part disappears when appropriate zero-bin contributions are subtracted, not from the soft part.

In fact, we do not have to relate the δ regulators to the offshellness of the particles. The δ regulators may be used also in the soft Wilson lines, but we stress that the δ regulators in the collinear and soft Wilson lines are independent regulators. They can be related only when

we obtain the δ regulators from the offshellness in the back-to-back current. Otherwise, the attempt to assign some relations between them may simplify the computation, but it evades the consistent treatment of the rapidity divergence.

The total collinear contribution including the self-energy corrections with the soft zero-bin subtraction is given at one loop as

$$\begin{aligned}
M_{\text{col}} &= 2(M_a - M_a^0) + 2(M_b - M_{b,\text{soft}}^0) + M_c + (Z_\xi + R_\xi)\delta(1-x)\frac{\delta(\mathbf{k}_\perp^2)}{\pi} \\
&= \frac{\alpha_s C_F}{2\pi^2} \left\{ \delta(\mathbf{k}_\perp^2) \left[\delta(1-x) \left(\frac{2}{\epsilon^2} + \frac{1}{\epsilon} \left(\frac{3}{2} + 2 \ln \frac{\mu^2}{-p^2} \right) + \frac{7}{2} - \frac{\pi^2}{2} + \frac{3}{2} \ln \frac{\mu^2}{-p^2} \right) \right. \right. \\
&\quad \left. \left. + \frac{1+x^2}{(1-x)_+} \left(\ln \frac{\mu^2}{-p^2} - \ln x \right) - (1+x^2) \frac{\ln(1-x)}{(1-x)_+} \right] - (1-x) \right. \\
&\quad \left. + \left(\frac{1+x^2}{(1-x)_+} - 2\delta(1-x) \ln \frac{\mu^2}{-p^2} \right) \left[\frac{1}{\mathbf{k}_\perp^2} \right]_{\mu^2} - 2\delta(1-x) \left[\frac{1}{\mathbf{k}_\perp^2} \ln \frac{\mathbf{k}_\perp^2}{\mu^2} \right]_{\mu^2} \right\}, \quad (28)
\end{aligned}$$

where Z_ξ and R_ξ are the wave function renormalization and the residue for the collinear fermion ξ at order α_s . They are given by

$$Z_\xi = -\frac{\alpha_s C_F}{4\pi} \frac{1}{\epsilon}, \quad R_\xi = -\frac{\alpha_s C_F}{4\pi} \left(1 + \ln \frac{\mu^2}{-p^2} \right). \quad (29)$$

The collinear contribution, or the TMDPDF is independent of δ . That is, it is free of rapidity divergence. It is also true in the radiative corrections for the TMDPDF $f_{q/N}(x, \mathbf{k}_\perp^2)$. Note that the mixing between the UV and IR divergences still remains. The mixing is cancelled if the soft part is added.

IV. SOFT FUNCTION AND ITS ONE-LOOP CORRECTIONS

The soft function is defined as

$$S(\boldsymbol{\eta}_\perp) = \frac{1}{N_c} \langle 0 | \text{tr} \left[Y_n^\dagger Y_{\bar{n}} \delta^{(2)}(\boldsymbol{\eta}_\perp + i\nabla_\perp) Y_{\bar{n}}^\dagger Y_n \right] | 0 \rangle. \quad (30)$$

The soft Wilson lines with the offshellness to regulate IR divergence are given by

$$Y_n = \sum_{\text{perm}} \exp \left[-\frac{1}{n \cdot \mathcal{P} + \Delta_1 + i0} g n \cdot A_s \right], \quad Y_{\bar{n}} = \sum_{\text{perm}} \exp \left[-\frac{1}{\bar{n} \cdot \mathcal{P} + \Delta_2 + i0} g \bar{n} \cdot A_s \right]. \quad (31)$$

The operators $n \cdot \mathcal{P}$ and $\bar{n} \cdot \mathcal{P}$ extract the n - and \bar{n} -components of the momenta from the soft gluon. The appearance of Δ_1 and Δ_2 in Y_n and $Y_{\bar{n}}$ resembles the δ regulators in the collinear Wilson lines, but they are from different origins. Recall that the soft Wilson lines are obtained by integrating out the offshellness of the intermediate states when soft gluons are emitted from a collinear particle. Therefore, unlike the collinear Wilson lines, Δ_i 's depend solely on the offshellness of the relevant collinear particle, not other jets. And it should be noted that these regulators in the soft Wilson lines regulate the IR divergence, not the rapidity divergence. We can write $\Delta_1 = p_1^2/p_1^-$, $\Delta_2 = p_2^2/p_2^+$. Only in the case of the

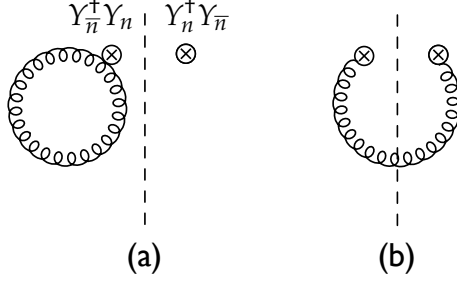


FIG. 2. Feynman diagrams for soft contributions (a) virtual corrections and (b) real gluon emission.

back-to-back current, the δ regulators in the collinear Wilson line are given by $\Delta_1 = \delta_2$ and $\Delta_2 = \delta_1$.

The Feynman diagrams for the radiative correction of $S(\boldsymbol{\eta}_\perp)$ at one loop are shown in Fig. 2. The virtual correction in Fig. 2 (a), and the real gluon emission in Fig. 2 (b) with their hermitian conjugates are given by

$$\begin{aligned} M_a^s &= \frac{\alpha_s C_F}{2\pi^2} \delta(\boldsymbol{\eta}_\perp^2) \left(-\frac{2}{\epsilon^2} - \frac{2}{\epsilon} \ln \frac{\mu^2}{\Delta_1 \Delta_2} - \ln^2 \frac{\mu^2}{\Delta_1 \Delta_2} + \frac{\pi^2}{2} \right), \\ M_b^s &= \frac{\alpha_s C_F}{2\pi^2} \left[\delta(\boldsymbol{\eta}_\perp^2) \left(\ln^2 \frac{\mu^2}{\Delta_1 \Delta_2} - \frac{\pi^2}{3} \right) + 2 \left[\frac{1}{\boldsymbol{\eta}_\perp^2} \ln \frac{\boldsymbol{\eta}_\perp^2}{\Delta_1 \Delta_2} \right]_{\mu^2} \right], \end{aligned} \quad (32)$$

and the total soft contribution is given by

$$M_{\text{soft}} = M_a^s + M_b^s = \frac{\alpha_s C_F}{2\pi^2} \left[\delta(\boldsymbol{\eta}_\perp^2) \left(-\frac{2}{\epsilon^2} - \frac{2}{\epsilon} \ln \frac{\mu^2}{\Delta_1 \Delta_2} + \frac{\pi^2}{6} \right) + \left[\frac{2}{\boldsymbol{\eta}_\perp^2} \ln \frac{\boldsymbol{\eta}_\perp^2}{\Delta_1 \Delta_2} \right]_{\mu^2} \right]. \quad (33)$$

The soft function depends on the IR regulators Δ_i . Especially, there is mixing between the UV and IR divergences. When it is combined with the collinear contributions in Eq. (28) with $p^2 = p_1^2$, and another collinear contribution from the TMDPDF in the \bar{n} direction with $p^2 = p_2^2$, the mixing of the UV and IR divergences cancels out.

V. TMDPDF AND INTEGRATED PDF

The integrated PDF is defined also in terms of the collinear fields only as

$$\phi_{q/N}\left(\frac{\omega}{p_-}\right) = \langle N(p^-) | \bar{\chi}_n \frac{\not{\bar{n}}}{2} \delta(\omega - \bar{n} \cdot \mathcal{P}) \chi_n | N(p^-) \rangle. \quad (34)$$

Compared to the definition of the TMDPDF, the only difference is the absence of the two-dimensional delta function on the transverse momentum. That is, the integrated PDF can be obtained from TMDPDF by integrating over the transverse momentum. It looks trivial to obtain the relation between the TMDPDF and the integrated PDF, but it is not trivial to verify the relation at higher orders [14]. The complication arises due to the regularization methods employed to tame the UV, IR and rapidity divergences. The UV divergence can be

typically treated in dimensional regularization. The rapidity divergence is handled by the δ regulator in the collinear Wilson line with the soft zero-bin subtraction. The IR divergence is taken care of only when the collinear and soft parts are summed. In this procedure, the collinear and soft parts are entangled because the regularization methods complicate the transparent relation between the TMDPDF and the integrated PDF, and it needs special care to see the relation at higher orders.

The important issue in establishing the relation is the range of the transverse momentum. When the transverse momentum is to be integrated, the region of integration should be determined beforehand. If we confine ourselves to finite transverse momentum, there is no UV divergence in real gluon emissions. Since the Drell-Yan process with the transverse momentum $q_T \sim Q\lambda$ is considered, we can limit the region of validity accordingly. In that case, we use the idea of μ^2 distribution which is suggested exactly for this purpose. On the other hand, the transverse momentum can extend to infinity in the effective theory, and the μ^2 distribution is not valid any more. There are two possible ways to establish the relation between the TMDPDF and the integrated PDF. First, the radiative corrections for the integrated PDF are performed in the usual way, and we modify the μ^2 distribution in the TMDPDF to include the infinite transverse momentum. Secondly, in the spirit of computing TMDPDF assuming that the transverse momentum never reaches infinity but some finite scale Λ_T , the radiative corrections of the integrated PDF are modified such that the integration over the transverse momentum is limited to Λ_T in real gluon emissions, while the virtual corrections are performed as usual.

Let us first consider the case in which the transverse momentum can go to infinity. The radiative corrections to the integrated PDF can be computed using the Feynman diagrams in Fig. 1 without the two-dimensional delta function for the transverse momentum. We organize the collinear matrix elements for integrated PDF including the zero-bin subtractions in terms of the virtual and real gluon emissions as

$$\begin{aligned}
M_{\text{PDF}}^V &= 2\tilde{M}_a + (Z_\xi + R_\xi)\delta(1-x)\frac{\delta(\mathbf{k}_\perp^2)}{\pi} \\
&= \frac{\alpha_s C_F}{2\pi}\delta(1-x)\left(\frac{2}{\epsilon^2} + \frac{3}{2\epsilon} + \frac{2}{\epsilon}\ln\frac{\mu^2}{-p^2} + \frac{3}{2}\ln\frac{\mu^2}{-p^2} + \ln^2\frac{\mu^2}{-p^2} + \frac{7}{2} - \frac{\pi^2}{6}\right), \\
M_{\text{PDF}}^R &= 2\tilde{M}_b + M_c \\
&= \frac{\alpha_s C_F}{2\pi}\left[\delta(1-x)\left(-\frac{2}{\epsilon^2} - \frac{2}{\epsilon}\ln\frac{\mu^2}{-p^2} - \ln^2\frac{\mu^2}{-p^2} - \frac{\pi^2}{6}\right)\right. \\
&\quad \left.+ \frac{1+x^2}{(1-x)_+}\left(\frac{1}{\epsilon} + \ln\frac{\mu^2}{-p^2} - \ln x\right) - (1+x^2)\left(\frac{\ln(1-x)}{1-x}\right)_+ - 2(1-x)\right]. \tag{35}
\end{aligned}$$

And the total collinear contributions at one loop are given as

$$\begin{aligned}
M_{\text{PDF}} &= M_{\text{PDF}}^V + M_{\text{PDF}}^R \\
&= \frac{\alpha_s C_F}{2\pi}\left[\delta(1-x)\left(\frac{3}{2\epsilon} + \frac{3}{2}\ln\frac{\mu^2}{-p^2} + \frac{7}{2} - \frac{\pi^2}{3}\right) - 2(1-x)\right. \\
&\quad \left.+ \frac{1+x^2}{(1-x)_+}\left(\frac{1}{\epsilon} + \ln\frac{\mu^2}{-p^2} - \ln x\right) - (1+x^2)\left(\frac{\ln(1-x)}{1-x}\right)_+\right]. \tag{36}
\end{aligned}$$

The result is independent of the δ regulators, which means that there is no rapidity divergence in the integrated PDF.

For the TMDPDF, we need another distribution called the “infinity” distribution, in which the finite scale Λ_T in defining the μ^2 distribution extends to infinity. The function of the form

$$g_1(\mathbf{k}_\perp^2, \epsilon, \Delta) = \frac{\mu^{2\epsilon}}{(\mathbf{k}_\perp^2 + \Delta)^{1+\epsilon}}, \quad (37)$$

appears in the one-loop correction, where Δ is a small quantity. For a regular test function $f(\mathbf{k}_\perp^2)$, the integral of $f(\mathbf{k}_\perp^2)$ multiplied by $g_1(\mathbf{k}_\perp^2, \epsilon, \Delta)$ is written as

$$\begin{aligned} \int_0^\infty d\mathbf{k}_\perp^2 g_1(\mathbf{k}_\perp^2, \epsilon, \Delta) f(\mathbf{k}_\perp^2) &= \int_0^\infty d\mathbf{k}_\perp^2 g_1(\mathbf{k}_\perp^2, \epsilon, 0) (f(\mathbf{k}_\perp^2) - f(0)) + f(0) \int_0^\infty d\mathbf{k}_\perp^2 g_1(\mathbf{k}_\perp^2, \epsilon, \Delta) \\ &= \int_0^\infty d\mathbf{k}_\perp^2 [g_1(\mathbf{k}_\perp^2, \epsilon, 0)]_\infty f(\mathbf{k}_\perp^2) + f(0) \left(\frac{1}{\epsilon} + \ln \frac{\mu^2}{\Delta} \right). \end{aligned} \quad (38)$$

In the infinity distribution function, we put $\Delta = 0$ since the IR divergence is already regulated. Compared to the μ^2 distribution function, the only difference is the appearance of the pole $1/\epsilon$. This comes from the UV region, which cannot be reached in the μ^2 distribution function. Therefore, $g_1(\mathbf{k}_\perp^2, \epsilon, \Delta)$ can be written as

$$g_1(\mathbf{k}_\perp^2, \epsilon, \Delta) = \left(\frac{1}{\epsilon} + \ln \frac{\mu^2}{\Delta} \right) \delta(\mathbf{k}_\perp^2) + [g_1(\mathbf{k}_\perp^2, \epsilon, 0)]_\infty. \quad (39)$$

Similarly, there is another function expressed in terms of the infinity distribution functions as

$$\begin{aligned} g_2(\mathbf{k}_\perp^2, \epsilon, \Delta) &= \frac{\mu^{2\epsilon} \ln \frac{\mathbf{k}_\perp^2}{\Delta}}{(\mathbf{k}_\perp^2 - \Delta)^{1+\epsilon}} \\ &= \left(\frac{1}{\epsilon^2} + \frac{1}{\epsilon} \ln \frac{\mu^2}{\Delta} + \frac{1}{2} \ln^2 \frac{\mu^2}{\Delta} + \frac{\pi^2}{3} \right) \delta(\mathbf{k}_\perp^2) + \left[\frac{1}{\mathbf{k}_\perp^2} \right]_\infty \ln \frac{\mu^2}{\Delta} + \left[\frac{\ln \mathbf{k}_\perp^2 / \mu^2}{\mathbf{k}_\perp^2} \right]_\infty. \end{aligned} \quad (40)$$

Here two points should be noted. First, the terms proportional to $\delta(\mathbf{k}_\perp^2)$ in the μ^2 distribution have the same μ dependence as those in the infinity distribution except the UV poles. And secondly, the infinity distribution is not exactly a distribution function in a rigorous sense. It is because the UV pole cannot be peaked near $\mathbf{k}_\perp^2 \sim 0$, therefore it should be understood that the infinity distribution function is meaningful only after the integral over \mathbf{k}_\perp^2 is performed. This awkward situation occurs since we are going to compare the TMDPDF and the integrated PDF, and the integrated PDF is obtained by integrating \mathbf{k}_\perp^2 over all values including infinity.

The total radiative correction for the TMDPDF is given as

$$\begin{aligned} M_{\text{TMDPDF}} &= \frac{\alpha_s C_F}{2\pi^2} \left\{ \delta(\mathbf{k}_\perp^2) \left[\delta(1-x) \left(\frac{3}{2\epsilon} + \frac{7}{2} - \frac{\pi^2}{3} + \frac{3}{2} \ln \frac{\mu^2}{-p^2} \right) - 2(1-x) \right. \right. \\ &\quad \left. \left. + \frac{1+x^2}{(1-x)_+} \left(\frac{1}{\epsilon} + \ln \frac{\mu^2}{-p^2} - \ln x \right) - (1+x^2) \left(\frac{\ln(1-x)}{1-x} \right)_+ \right] \right. \\ &\quad \left. + \frac{1+x^2}{(1-x)_+} \left[\frac{1}{\mathbf{k}_\perp^2} \right]_\infty - 2\delta(1-x) \left[\frac{1}{\mathbf{k}_\perp^2} \ln \frac{\mathbf{k}_\perp^2}{-p^2} \right]_\infty \right\}. \end{aligned} \quad (41)$$

If we integrate over \mathbf{k}_\perp , the infinity distributions vanish and we obtain

$$\int d^2\mathbf{k}_\perp M_{\text{TMDPDF}} = \frac{\alpha_s C_F}{2\pi} \left[\delta(1-x) \left(\frac{3}{2\epsilon} + \frac{7}{2} - \frac{\pi^2}{3} + \frac{3}{2} \ln \frac{\mu^2}{-p^2} \right) - 2(1-x) \right. \\ \left. + \frac{1+x^2}{(1-x)_+} \left(\frac{1}{\epsilon} + \ln \frac{\mu^2}{-p^2} - \ln x \right) - (1+x^2) \left(\frac{\ln(1-x)}{1-x} \right)_+ \right], \quad (42)$$

which is exactly the same as the radiative corrections for the integrated PDF in Eq. (36).

Secondly, we consider the case in which the transverse momentum has the range $0 \leq \mathbf{k}_\perp^2 \leq \Lambda_T^2$. Then the radiative corrections for the integrated PDF is handled differently. The loop momentum l is defined in D dimensions, and there is a two-dimensional delta function. We introduce the momentum vector in $D-4 = -2\epsilon$ dimensions, \hat{l}_\perp , and \mathbf{l}_\perp is a two-dimensional momentum vector. Then l^2 can be written as $l^2 = l^+ l^- - \mathbf{l}_\perp^2 - \hat{l}_\perp^2$. First the two-dimensional transverse momentum is fixed in real gluon emissions and the integral over \hat{l}_\perp in $D-4$ dimensions is performed first. Then we integrate over the two-dimensional transverse momentum. Since the virtual corrections are not affected, \tilde{M}_a is the same. And the matrix elements for the real gluon emissions are modified to be

$$M_{\text{PDF}}^R = \frac{\alpha_s C_F}{2\pi} \left[\delta(1-x) \left(-\frac{\pi^2}{6} - \frac{1}{2} \ln^2 \frac{\Lambda_T^2}{-p^2} \right) + \frac{1+x^2}{(1-x)_+} \left(\ln \frac{\Lambda_T^2}{-p^2} - \ln x \right) \right. \\ \left. - (1+x^2) \left(\frac{\ln(1-x)}{1-x} \right)_+ - (1-x) \right]. \quad (43)$$

The whole collinear part of the integrated PDF at one loop is given by

$$M_{\text{PDF}} = \frac{\alpha_s C_F}{2\pi} \left\{ \delta(1-x) \left[\frac{2}{\epsilon^2} + \frac{1}{\epsilon} \left(\frac{3}{2} + 2 \ln \frac{\mu^2}{-p^2} \right) + \frac{7}{2} - \frac{\pi^2}{2} + \frac{3}{2} \ln \frac{\mu^2}{-p^2} + \ln^2 \frac{\mu^2}{-p^2} - \ln^2 \frac{\Lambda_T^2}{-p^2} \right] \right. \\ \left. + \frac{1+x^2}{(1-x)_+} \left(\ln \frac{\Lambda_T^2}{-p^2} - \ln x \right) - (1+x^2) \frac{\ln(1-x)}{(1-x)_+} - (1-x) \right\}. \quad (44)$$

If we integrate Eq. (28) from 0 to Λ_T^2 , Eq. (44) is obtained. In either case, the integrated PDF is consistently obtained by integrating the TMDPDF over the transverse momentum to one loop.

VI. DIFFERENTIAL SCATTERING CROSS SECTION AT NLO

The TMDPDF can be expressed in terms of the convolution of the integrated PDF and the TMD kernel which contains all the information on the transverse momentum in the TMDPDF. Its relation, Eq. (10), is given again as

$$f_{q/N}(x, \mathbf{k}_\perp, \mu) = \int_x^1 \frac{dz}{z} J_{qq}(z, \mathbf{k}_\perp, \mu) \phi_{q/N}\left(\frac{x}{z}, \mu\right) = \int_x^1 \frac{dz}{z} J_{qq}\left(\frac{x}{z}, \mathbf{k}_\perp, \mu\right) \phi_{q/N}(z, \mu), \quad (45)$$

where $\phi_{q/N}(z)$ is the integrated PDF. We will use this relation to obtain the renormalization group behavior of the scattering cross section to one loop. Eq. (45) can be regarded as matching between two SCET's, where the TMDPDF is the quantity in which $p \sim Q\lambda$ and

the integrated PDF is the one where $p \sim Q\lambda^2$. And the kernel $J_{qq}(z, \mathbf{k}_\perp, \mu)$ is the Wilson coefficient in matching. At zeroth order in α_s , $J_{qq}^{(0)}$ and $\phi_{q/N}^{(0)}$ are given by

$$J_{qq}^{(0)}(z, \mathbf{k}_\perp) = \frac{1}{\pi} \delta(1-z) \delta(\mathbf{k}_\perp^2), \quad \phi_{q/N}^{(0)}(z) = \delta(1-z). \quad (46)$$

The TMDPDF and the soft functions at order α_s are obtained from Eqs. (28) and (33) by removing the UV divergent terms. They seem to have the mixed divergent terms, but they disappear when all the contributions are added, therefore it is of no concern here. They are given as

$$\begin{aligned} f_{q/N}^{(1)}(x, \mathbf{k}_\perp, \mu) &= \frac{\alpha_s C_F}{2\pi^2} \left\{ \delta(\mathbf{k}_\perp^2) \left[\delta(1-x) \left(\frac{3}{2} \ln \frac{\mu^2}{-p^2} + \frac{7}{2} - \frac{\pi^2}{2} \right) \right. \right. \\ &\quad \left. \left. + \frac{1+x^2}{(1-x)_+} \left(\ln \frac{\mu^2}{-p^2} - \ln x \right) - (1+x^2) \left(\frac{\ln(1-x)}{1-x} \right)_+ - (1-x) \right] \right. \\ &\quad \left. - 2\delta(1-x) \left[\frac{1}{\mathbf{k}_\perp^2} \ln \frac{\mathbf{k}_\perp^2}{-p^2} \right]_{\mu^2} + \frac{1+x^2}{(1-x)_+} \left[\frac{1}{\mathbf{k}_\perp^2} \right]_{\mu^2} \right\}, \\ S^{(1)}(\boldsymbol{\eta}_\perp) &= \frac{\alpha_s C_F}{2\pi^2} \left\{ \frac{\pi^2}{6} \delta(\boldsymbol{\eta}_\perp^2) + 2 \left[\frac{1}{\boldsymbol{\eta}_\perp^2} \ln \frac{\boldsymbol{\eta}_\perp^2}{\Delta_1 \Delta_2} \right]_{\mu^2} \right\}. \end{aligned} \quad (47)$$

Expanding Eq. (45) to first order in α_s , we obtain

$$\begin{aligned} f_{q/N}^{(1)}(x, \mathbf{k}_\perp) &= \int_x^1 \frac{dz}{z} \left[J^{(1)}(z, \mathbf{k}_\perp) \phi_{q/N}^{(0)}(x/z) + J^{(0)}(x/z, \mathbf{k}_\perp) \phi_{q/N}^{(1)}(z) \right] \\ &= J^{(1)}(x, \mathbf{k}_\perp) + \frac{1}{\pi} \delta(\mathbf{k}_\perp^2) \phi_{q/N}^{(1)}(x). \end{aligned} \quad (48)$$

The integrated PDF at order α_s is given by

$$\begin{aligned} \phi_{q/N}^{(1)}(x) &= \frac{\alpha_s C_F}{2\pi} \left\{ \delta(1-x) \left(\frac{7}{2} - \frac{\pi^2}{3} + \frac{3}{2} \ln \frac{\mu^2}{-p^2} \right) - 2(1-x) \right. \\ &\quad \left. + \frac{1+x^2}{(1-x)_+} \left(\ln \frac{\mu^2}{-p^2} - \ln x \right) - (1+x^2) \left(\frac{\ln(1-x)}{1-x} \right)_+ \right\}, \end{aligned} \quad (49)$$

and the anomalous dimension for the PDF is given by

$$\gamma_\phi(x) = \frac{\alpha_s}{2\pi} P_{qq}(x) = \frac{\alpha_s C_F}{2\pi} \left[\frac{3}{2} \delta(1-z) + \frac{1+z^2}{(1-z)_+} \right]. \quad (50)$$

Using Eqs. (48) and (49), the new result is the TMD kernel at order α_s , which is given as

$$\begin{aligned} J^{(1)}(x, \mathbf{k}_\perp, \mu) &= f_{q/N}^{(1)}(x, \mathbf{k}_\perp, \mu) - \frac{1}{\pi} \delta(\mathbf{k}_\perp^2) \phi_{q/N}^{(1)}(x, \mu) \\ &= \frac{\alpha_s C_F}{2\pi^2} \left\{ \delta(\mathbf{k}_\perp^2) \left(1-x - \frac{\pi^2}{6} \right) + \frac{1+x^2}{(1-x)_+} \left[\frac{1}{\mathbf{k}_\perp^2} \right]_{\mu^2} - 2\delta(1-x) \left[\frac{1}{\mathbf{k}_\perp^2} \ln \frac{\mathbf{k}_\perp^2}{-p^2} \right]_{\mu^2} \right\}, \end{aligned} \quad (51)$$

where $[f(\mathbf{k}_\perp^2)]_{\mu^2}$ is defined as

$$[f(\mathbf{k}_\perp^2)]_{\mu^2} = f(\mathbf{k}_\perp^2) - \delta(\mathbf{k}_\perp^2) \int_0^{\mu^2} d\mathbf{l}_\perp^2 f(\mathbf{l}_\perp^2). \quad (52)$$

The factorized differential scattering cross section is written as

$$\begin{aligned}
\frac{d\sigma}{d^2\mathbf{q}_\perp} &= \int dx_1 dx_2 \sigma_0 H(Q, \mu) \int d^2\mathbf{l}_{1\perp} d^2\mathbf{l}_{2\perp} d^2\boldsymbol{\eta}_\perp \delta^{(2)}(\mathbf{q}_\perp - \mathbf{l}_{1\perp} - \mathbf{l}_{2\perp} - \boldsymbol{\eta}_\perp) S(\boldsymbol{\eta}_\perp) \\
&\times \int_{x_1}^1 \frac{dz_1}{z_1} J_{qq}(z_1, \mathbf{l}_{1\perp}) \phi_{q/N_1}\left(\frac{x_1}{z_1}\right) \int_{x_2}^1 \frac{dz_2}{z_2} J_{\bar{q}\bar{q}}(z_2, \mathbf{l}_{2\perp}) \phi_{\bar{q}/N_2}\left(\frac{x_2}{z_2}\right) \\
&= \int dx_1 dx_2 \sigma_0 H(Q, \mu) \int_{x_1}^1 \frac{dz_1}{z_1} \int_{x_2}^1 \frac{dz_2}{z_2} W(z_1, z_2, \mathbf{q}_\perp, \mu) \phi_{q/N_1}\left(\frac{x_1}{z_1}, \mu\right) \phi_{\bar{q}/N_2}\left(\frac{x_2}{z_2}, \mu\right).
\end{aligned} \tag{53}$$

To NLO, the TMD kernel $W(z_1, z_2, \mathbf{q}_\perp, \mu)$ can be written as

$$\begin{aligned}
W(z_1, z_2, \mathbf{q}_\perp) &= \int d^2\mathbf{l}_{1\perp} d^2\mathbf{l}_{2\perp} d^2\boldsymbol{\eta}_\perp \delta^{(2)}(\mathbf{q}_\perp - \mathbf{l}_{1\perp} - \mathbf{l}_{2\perp} - \boldsymbol{\eta}_\perp) S(\boldsymbol{\eta}_\perp) J_{qq}(z_1, \mathbf{l}_{1\perp}) J_{\bar{q}\bar{q}}(z_2, \mathbf{l}_{2\perp}) \\
&= \int d^2\mathbf{l}_{1\perp} d^2\mathbf{l}_{2\perp} d^2\boldsymbol{\eta}_\perp \delta^{(2)}(\mathbf{q}_\perp - \mathbf{l}_{1\perp} - \mathbf{l}_{2\perp} - \boldsymbol{\eta}_\perp) \\
&\times \left[S^{(1)}(\boldsymbol{\eta}_\perp) J_{qq}^{(0)}(z_1, \mathbf{l}_{1\perp}) J_{\bar{q}\bar{q}}^{(0)}(z_2, \mathbf{l}_{2\perp}) + S^{(0)}(\boldsymbol{\eta}_\perp) J_{qq}^{(1)}(z_1, \mathbf{l}_{1\perp}) J_{\bar{q}\bar{q}}^{(0)}(z_2, \mathbf{l}_{2\perp}) \right. \\
&\left. + S^{(0)}(\boldsymbol{\eta}_\perp) J_{qq}^{(0)}(z_1, \mathbf{l}_{1\perp}) J_{\bar{q}\bar{q}}^{(1)}(z_2, \mathbf{l}_{2\perp}) \right] \\
&= \frac{\alpha_s C_F}{2\pi^2} \left\{ \delta(\mathbf{q}_\perp^2) \left[\delta(1-z_1)(1-z_2) + \delta(1-z_2)(1-z_1) - \frac{\pi^2}{6} \delta(1-z_1)\delta(1-z_2) \right] \right. \\
&\quad + \left[\frac{2}{\mathbf{q}_\perp^2} \ln \frac{Q^2}{\mathbf{q}_\perp^2} \right]_{\mu^2} \delta(1-z_1)\delta(1-z_2) \\
&\quad \left. + \left[\frac{1}{\mathbf{q}_\perp^2} \right]_{\mu^2} \left[\frac{1+z_1^2}{(1-z_1)_+} \delta(1-z_2) + \delta(1-z_1) \frac{1+z_2^2}{(1-z_2)_+} \right] \right\}.
\end{aligned} \tag{54}$$

The anomalous dimension of W at one loop is given as

$$\begin{aligned}
\gamma_W(z_1, z_2, \mathbf{q}_\perp^2, \mu) &= \frac{d}{d \ln \mu} W(z_1, z_2, \mathbf{q}_\perp, \mu) = 2\mu^2 \frac{d}{d\mu^2} W(z_1, z_2, \mathbf{q}_\perp, \mu) \\
&= \frac{\alpha_s C_F}{2\pi} \frac{\delta(\mathbf{q}_\perp^2)}{\pi} \left[2 \ln \frac{\mu^2}{Q^2} \delta(1-z_1)\delta(1-z_2) \right. \\
&\quad \left. - \frac{1+z_1^2}{(1-z_1)_+} \delta(1-z_2) - \delta(1-z_1) \frac{1+z_2^2}{(1-z_2)_+} \right] \\
&= \frac{\alpha_s}{2\pi} \frac{\delta(\mathbf{q}_\perp^2)}{\pi} \left[C_F \left(2 \ln \frac{\mu^2}{Q^2} + 3 \right) \delta(1-z_1)\delta(1-z_2) \right. \\
&\quad \left. - P_{qq}(z_1)\delta(1-z_2) - P_{\bar{q}\bar{q}}(z_2)\delta(1-z_1) \right],
\end{aligned} \tag{55}$$

where $P_{qq}(z) = P_{\bar{q}\bar{q}}(z)$. It can be also written as

$$\begin{aligned}
\gamma_W(z_1, z_2, \mathbf{q}_\perp^2, \mu) &= \delta(\mathbf{q}_\perp^2) \tilde{\gamma}_W(z_1, z_2, \mu) \\
&= \gamma_{J_{qq}}(z_1, \mathbf{q}_\perp^2, \mu) \delta(1-z_1) + \gamma_{J_{\bar{q}\bar{q}}}(z_2, \mathbf{q}_\perp^2, \mu) \delta(1-z_2) \\
&\quad + \gamma_S(\mathbf{q}_\perp^2) \delta(1-z_1) \delta(1-z_2).
\end{aligned} \tag{56}$$

Here $\gamma_{J_{qq}}$, $\gamma_{J_{\bar{q}\bar{q}}}$ and γ_S are the anomalous dimensions for J_{qq} , $J_{\bar{q}\bar{q}}$ and the soft function, which are given as

$$\begin{aligned}
\gamma_J(x, \mathbf{q}_\perp^2, \mu) &= \frac{d}{d \ln \mu} J(x, \mathbf{q}_\perp, \mu) = \frac{\alpha_s C_F}{2\pi} \frac{\delta(\mathbf{q}_\perp^2)}{\pi} \left[2\delta(1-x) \ln \frac{\mu^2}{-p^2} - \frac{1+x^2}{(1-x)_+} \right], \\
\gamma_S(\mathbf{q}_\perp^2, \mu) &= \frac{d}{d \ln \mu} S(\mathbf{q}_\perp, \mu) = -2 \frac{\alpha_s C_F}{\pi} \frac{\delta(\mathbf{q}_\perp^2)}{\pi} \ln \frac{\mu^2}{\Delta_1 \Delta_2},
\end{aligned} \tag{57}$$

where $\Delta_1 = p_1^2/\bar{n} \cdot p_1$, and $\Delta_2 = p_2^2/n \cdot p_2$, and $J = J_{q\bar{q}} = J_{\bar{q}q}$. Note that γ_J and γ_S still depend on the IR-dependent terms, but the sum γ_W has no IR divergence. Compared to the anomalous dimensions corresponding to γ_J and γ_S in Ref. [16], the dependence on the rapidity scale ν is tantamount to the IR dependence in our approach.

The hard coefficient $C(Q, \mu)$ is given by

$$C(Q, \mu) = 1 + \frac{\alpha_s C_F}{4\pi} \left(-\ln^2 \frac{\mu^2}{-Q^2} - 3 \ln \frac{\mu^2}{-Q^2} - 8 + \frac{\pi^2}{6} \right), \quad (58)$$

and the anomalous dimension of the hard part $H(Q, \mu) = |C(Q, \mu)|^2$ is given as

$$\gamma_H(\mu) = -\frac{\alpha_s C_F}{\pi} \left(2 \ln \frac{\mu^2}{Q^2} + 3 \right). \quad (59)$$

Combining γ_W , γ_H and γ_ϕ , it is clear that the factorized scattering cross section, Eq. (53), is independent of the renormalization scale μ at one loop. The numerical analysis for the scattering cross sections is interesting with the solution for the renormalization group equation. This can be applied to the processes with small transverse momentum such as Drell-Yan process, Z boson production as well as Higgs production both at Tevatron and LHC. The numerical estimates involving TMDPDF in these processes will be presented in future publication.

VII. DECOUPLING OF THE UV AND IR DIVERGENCES

In order to see how the decoupling of the UV and IR divergences is achieved in various combinations, it is convenient to collect all the results obtained so far. The results for the integrated PDF are summarized as follows: The collinear parts are separated into the virtual and real parts as

$$\begin{aligned} M_{\text{PDF}}^V(p_1^2) &= \frac{\alpha_s C_F}{2\pi} \delta(1-x) \left[\frac{2}{\epsilon^2} + \frac{3}{2\epsilon} + \frac{2}{\epsilon} \ln \frac{\mu^2}{-p_1^2} + \frac{3}{2} \ln \frac{\mu^2}{-p_1^2} + \ln^2 \frac{\mu^2}{-p_1^2} + \frac{7}{2} - \frac{\pi^2}{6} \right], \\ M_{\text{PDF}}^R(p_1^2) &= \frac{\alpha_s C_F}{2\pi} \left[\delta(1-x) \left(-\frac{2}{\epsilon^2} - \frac{2}{\epsilon} \ln \frac{\mu^2}{-p_1^2} - \ln^2 \frac{\mu^2}{-p_1^2} - \frac{\pi^2}{6} \right) - 2(1-x) \right. \\ &\quad \left. + \frac{1+x^2}{(1-x)_+} \left(\frac{1}{\epsilon} - \ln x + \ln \frac{\mu^2}{-p_1^2} \right) + (1+x^2) \left(\frac{\ln(1-x)}{1-x} \right)_+ \right]. \end{aligned} \quad (60)$$

Note that there is another collinear contribution from the \bar{n} direction, which is obtained from replacing p_1^2 by p_2^2 . Similarly, the soft function is given as

$$\begin{aligned} M_{\text{soft}}^V &= \frac{\alpha_s C_F}{2\pi} \left(-\frac{2}{\epsilon^2} - \frac{2}{\epsilon} \ln \frac{\mu^2}{\Delta_1 \Delta_2} - \ln^2 \frac{\mu^2}{\Delta_1 \Delta_2} - \frac{\pi^2}{2} \right), \\ M_{\text{soft}}^R &= \frac{\alpha_s C_F}{2\pi} \left(\frac{2}{\epsilon^2} + \frac{2}{\epsilon} \ln \frac{\mu^2}{\Delta_1 \Delta_2} + \ln^2 \frac{\mu^2}{\Delta_1 \Delta_2} + \frac{\pi^2}{2} \right). \end{aligned} \quad (61)$$

The mixing terms $(\ln \mu^2/(-p^2))/\epsilon$ exists in M_{PDF}^V , M_{PDF}^R , M_{soft}^V and M_{soft}^R respectively. However, if we add the collinear parts $M_{\text{PDF}}^V + M_{\text{PDF}}^R$, the mixing term cancels. It is also true for

the soft parts $M_{\text{soft}}^V + M_{\text{soft}}^R$. Therefore the total contribution is free of the mixing. There is another combination in which the mixing cancels. They are $M_{\text{PDF}}^V(p_1^2) + M_{\text{PDF}}^V(p_2^2) + M_{\text{soft}}^V$, and $M_{\text{PDF}}^R(p_1^2) + M_{\text{PDF}}^R(p_2^2) + M_{\text{soft}}^R$. It means that the virtual corrections of the collinear and soft parts do not contain the mixing of the UV and IR divergences, nor the real gluon emissions of the collinear and soft parts.

In the case of the TMDPDF, the virtual corrections of the collinear and soft parts have the same expression except $\delta(\mathbf{k}_\perp^2)$, hence the mixing is not there. In the real gluon emissions, the possible mixing terms appear when \mathbf{k}_\perp^2 approaches infinity (or Λ_T^2) in the terms

$$\begin{aligned} M_{\text{TMDPDF}}^R &= 2\tilde{M}_b + M_c \rightarrow -2\left(\ln \frac{\mu^2}{-p_1^2} + \ln \frac{\mu^2}{-p_2^2}\right) \left[\frac{1}{\mathbf{k}_\perp^2}\right]_{\mu^2} \\ M_{\text{soft}}^R &= M_a^s \rightarrow 2\left(\ln \frac{\mu^2}{-\Delta_1} + \ln \frac{\mu^2}{-\Delta_2}\right) \left[\frac{1}{\boldsymbol{\eta}_\perp^2}\right]_{\mu^2}, \end{aligned} \quad (62)$$

where $\Delta_1 = p_1^2/\bar{n} \cdot p_1$ and $\Delta_2 = p_2^2/n \cdot p_2$. Other terms may contain divergences, but the UV and IR divergences are decoupled. As can be seen clearly, these are cancelled when the real gluon emissions in the collinear and soft parts are added. The mixing of the UV and IR divergences disappears in the virtual corrections and in the real gluon emissions of the collinear and soft parts respectively.

This observation is useful since the virtual corrections of the TMDPDF have the same form as those of the integrated PDF except $\delta(\mathbf{k}_\perp^2)$. And they do not have the mixing term. Therefore the real gluon emissions should not have any mixing term either, when integrated over the transverse momentum. As can be seen in Eq. (62), the mixing term is cancelled before the integration over the transverse momentum. If it is true to all orders in α_s , we can safely use this fact in order to obtain the anomalous dimension of the combined collinear and soft parts.

VIII. CONCLUSIONS

The divergence structure in the TMDPDF is intricate because the rapidity divergence should be also handled as well as the usual UV and IR divergences. One of the major problems in studying TMDPDF is to find a regularization method to treat the rapidity divergence. The dimensional regularization can handle the UV divergence, and possibly the IR divergence, but it cannot isolate the rapidity divergence. As far as the UV and IR divergences are concerned, it is convenient to use dimensional regularization to extract the UV divergence, and to employ the offshellness of external particles. There are various ways to treat rapidity divergence, and we have found that a simple way to regulate the rapidity divergence is achieved by inserting the δ regulators in the collinear Wilson lines only. Individual collinear Feynman diagrams depend on the regulator δ , but the total collinear contribution with the zero-bin subtraction is independent of the regulator, hence no rapidity divergence. The regulators similar to δ may be inserted, say, in the soft Wilson lines, or in the collinear fermion propagator, but they regulate the IR divergence, not the rapidity divergence.

For comparison, let us recapitulate previous approaches in handling the rapidity divergence. Collins and Soper [12] tilted the lightcone such that the rapidity divergence is handled by an extra scale in association with this tilting. Chiu et al. [16] modified the collinear and soft Wilson lines by introducing a rapidity scale ν , which treats the rapidity divergence. In both approaches, the hard, collinear and soft parts depend on the additional scale, while the cross sections do not. Therefore another renormalization group equation with respect to this scale is introduced. Another approach is to choose the axial gauge [28, 29], for example $\bar{n} \cdot A_n = 0$, instead of covariant gauges such as the Feynman gauge we chose. The advantage of this gauge choice is that many Feynman diagrams from the Wilson lines vanish, but there should be additional transverse gauge link which links the operators for TMDPDF at infinity so that it becomes gauge invariant. Ref. [23] treats the rapidity divergence in a similar way to ours, but they insert the δ regulators in both the collinear and the soft Wilson lines and give a specific relation between the collinear and the soft regulators. That relation, as we point out, holds only for the back-to-back current.

The separation of the UV and IR divergences is achieved in the sum of the virtual and real corrections in the collinear sector and in the soft sector, and in the sum of the collinear and soft parts in the virtual corrections and in the real gluon emissions. In effective theories like SCET, the mixing shows up often in intermediate steps, but it cancels in the final result. In the radiative correction of the TMDPDF, it also holds in the sums mentioned above. Therefore the renormalization group scaling can be considered only for the hard part and the sum of the collinear and soft parts. This prescription seems to violate the factorization in its strictest sense that the collinear and the soft parts are separately well defined. We rather claim that the attempt to combine the collinear part with part of the soft part such that the UV and IR divergences are decoupled is futile in its strictest sense that no such thing occurs with general regulators unless a very specific set of relations is imposed. The soft part interacts with all the collinear sectors in the process, hence contains the information of all the collinear sectors through their offshellness. Therefore it is physically more reasonable to consider the scaling behavior of the sum of the collinear and soft parts in order to decouple the UV and IR divergences. This may sound cumbersome, since it means that we have to compute all the collinear and soft parts first in complicated processes with multijets or heavy colored particles. But we stress that it is the correct procedure to treat high-energy scattering. It will be interesting to see if our procedure also works in other processes involving small transverse momentum. Various aspects of our formalism will be investigated in future publications.

The important result in this work is the factorization formula for the differential scattering cross section in Drell-Yan process with small transverse momentum, given by Eq. (11). The scattering cross section is factorized into the hard part, the TMD kernel and the PDF's. The hard part is the matching coefficient between the full theory and the SCET with the scale $Q\lambda$. The TMD kernel is obtained by matching the two SCET's with the scales $Q\lambda$ and $Q\lambda^2$. Therefore the TMD kernel should be free from IR divergence to all orders, and it is shown explicitly at NLO. Note that the TMD kernel $W(z_1, z_2, \mathbf{q}_\perp, \mu)$ is expressed in momentum space, rather than in impact parameter space. At least at NLO, the anomalous dimension of W is proportional to $\delta(\mathbf{q}_\perp^2)$, hence the dependence of the scaling behavior on

the transverse momentum is trivial.

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Appendix: μ^2 distribution functions and infinity distribution functions

Suppose that a function $g(x)$ diverges as $x \rightarrow 0$. Then the plus distribution function $[g(x)]_b$ can be defined, in general, as

$$\begin{aligned} \int_0^a [g(x)]_b f(x) dx &= \int_0^a dx g(x) f(x) - f(0) \int_0^b dx g(x) \\ &= \int_0^a g(x) [f(x) - f(0)] - f(0) \int_a^b dx g(x), \end{aligned} \quad (\text{A.1})$$

for a regular test function $f(x)$. The need for the plus distribution function is to separate the IR divergent part. When $g(x)$ diverges as $x \rightarrow 0$, the integral is tamed to be finite by subtracting $f(0)$. In this general definition of the plus distribution function, the upper limits a and b are arbitrary as long as the lower limits of the two integrals in Eq. (A.1) are both zero to guarantee that the IR divergence at $x = 0$ is cancelled in this subtraction. The finite remainder, the second term in Eq. (A.1), depends on the limits a and b . In a special case $a = b = 1$, the familiar plus distribution function is obtained as

$$\int_0^1 [g(x)]_+ f(x) = \int_0^1 dx g(x) [f(x) - f(0)]. \quad (\text{A.2})$$

There are two types of functions in the radiative corrections, which are of the form

$$g_1(\mathbf{k}_\perp^2, \epsilon, \delta) = \frac{\mu^{2\epsilon}}{(\mathbf{k}_\perp^2 + \Delta)^{1+\epsilon}}, \quad g_2(\mathbf{k}_\perp^2, \epsilon, \delta) = \frac{\mu^{2\epsilon} \ln \frac{\mathbf{k}_\perp^2}{\Delta}}{(\mathbf{k}_\perp^2 - \Delta)^{1+\epsilon}}, \quad (\text{A.3})$$

where Δ approaches zero. If $\Delta = 0$, these functions are singular at $\mathbf{k}_\perp^2 = 0$, that is, IR divergent. We extract the IR divergence as the coefficient of $\delta(\mathbf{k}_\perp^2)$, and the remainder is the distribution function free of the divergence. With the presence of the nonzero Δ , the IR divergence is regulated by Δ instead. However, there is a big difference in considering the distribution functions g_1 and g_2 in \mathbf{k}_\perp^2 -space compared to $[g(x)]_b$ since the upper limit can reach infinity. Irrespective of whether the upper limit goes to infinity or not, g_1 and g_2 diverge at $\mathbf{k}_\perp^2 = 0$ with $\Delta = 0$. If the upper limit in considering the integral including g_1 or g_2 reaches infinity, the integral also has an UV divergence. Therefore we classify the distribution functions according to the upper limits. These two cases have their own

physical importance. If we restrict the transverse momentum to be of order $Q\lambda$ and collect experimental data, there is finite upper limit which we call Λ_T . The relevant distribution function is called the “ μ^2 distribution”. In the effective theory, \mathbf{k}_\perp^2 can reach infinity and the UV divergence from the integration offers the information on the scaling behavior. The “infinity distribution” is devised for this case.

First let us consider the case in which the transverse momentum remains finite up to some scale Λ_T . If a regular test function $f(\mathbf{k}_\perp^2)$ multiplied by $g_1(\mathbf{k}_\perp^2, \epsilon, \Delta)$ is integrated from 0 to Λ_T , it yields

$$\begin{aligned}
& \int_0^{\Lambda_T^2} d\mathbf{k}_\perp^2 g_1(\mathbf{k}_\perp^2, \epsilon, \Delta) f(\mathbf{k}_\perp^2) \\
&= \int_0^{\Lambda_T^2} d\mathbf{k}_\perp^2 g_1(\mathbf{k}_\perp^2, \epsilon, \Delta) f(\mathbf{k}_\perp^2) - \int_0^{\mu^2} d\mathbf{k}_\perp^2 g_1(\mathbf{k}_\perp^2, \epsilon, \Delta) f(0) + \int_0^{\mu^2} d\mathbf{k}_\perp^2 g_1(\mathbf{k}_\perp^2, \epsilon, \Delta) f(0) \\
&= \int_0^{\Lambda_T^2} d\mathbf{k}_\perp^2 g_1(\mathbf{k}_\perp^2, 0, 0) f(\mathbf{k}_\perp^2) - \int_0^{\mu^2} d\mathbf{k}_\perp^2 g_1(\mathbf{k}_\perp^2, 0, 0) f(0) + f(0) \ln \frac{\mu^2}{\Delta} \\
&\equiv \int_0^{\Lambda_T^2} d\mathbf{k}_\perp^2 [g_1(\mathbf{k}_\perp^2, 0, 0)]_{\mu^2} f(\mathbf{k}_\perp^2) + f(0) \ln \frac{\mu^2}{\Delta}.
\end{aligned} \tag{A.4}$$

In the final expression, we put $\epsilon = 0$, and $\Delta = 0$ in $g(\mathbf{k}_\perp^2, \epsilon, \Delta)$ because the integral has no pole in ϵ , and the possible infrared divergence near $\mathbf{k}_\perp^2 \sim 0$ is cancelled in the subtraction. This defines the μ^2 distribution. g_2 can be treated in a similar way to yield

$$\begin{aligned}
g_1(\mathbf{k}_\perp^2, \epsilon, \Delta) &= \frac{\mu^{2\epsilon}}{(\mathbf{k}_\perp^2 + \Delta)^{1+\epsilon}} = \delta(\mathbf{k}_\perp^2) \ln \frac{\mu^2}{\Delta} + \left[\frac{1}{\mathbf{k}_\perp^2} \right]_{\mu^2}, \\
g_2(\mathbf{k}_\perp^2, \epsilon, \Delta) &= \frac{\mu^{2\epsilon} \ln \frac{\mathbf{k}_\perp^2}{\Delta}}{(\mathbf{k}_\perp^2 - \Delta)^{1+\epsilon}} = \delta(\mathbf{k}_\perp^2) \left(\frac{1}{2} \ln^2 \frac{\mu^2}{\Delta} + \frac{\pi^2}{3} \right) \\
&\quad + \left[\frac{\ln \mathbf{k}_\perp^2 / \mu^2}{\mathbf{k}_\perp^2} \right]_{\mu^2} + \left[\frac{1}{\mathbf{k}_\perp^2} \right]_{\mu^2} \ln \frac{\mu^2}{\Delta}.
\end{aligned} \tag{A.5}$$

Note that these functions are independent of μ since we can put $\epsilon = 0$ from the beginning. However, the μ dependence is split into the delta function and the μ^2 distribution function for convenience. The independence can be seen by computing the integral

$$\begin{aligned}
& \int_0^{\Lambda_T^2} d\mathbf{k}_\perp^2 g_1(\mathbf{k}_\perp^2, \epsilon, \Delta) f(\mathbf{k}_\perp^2) \\
&= \int_0^{\Lambda_T^2} d\mathbf{k}_\perp^2 [g_1(\mathbf{k}_\perp^2, 0, 0)]_{\mu^2} f(\mathbf{k}_\perp^2) + f(0) \ln \frac{\mu^2}{\Delta} \\
&= \int_0^{\Lambda_T^2} d\mathbf{k}_\perp^2 g_1(\mathbf{k}_\perp^2, 0, 0) [f(\mathbf{k}_\perp^2) - f(0)] + f(0) \ln \frac{\Lambda_T^2}{\Delta}.
\end{aligned} \tag{A.6}$$

If the upper limit for the transverse momentum reaches infinity, g_1 and g_2 contain UV divergences:

$$\begin{aligned}
\int_0^\infty d\mathbf{k}_\perp^2 g_1(\mathbf{k}_\perp^2, \epsilon, \Delta) &= \frac{1}{\epsilon} + \ln \frac{\mu^2}{\Delta} + \mathcal{O}(\epsilon) \\
\int_0^\infty d\mathbf{k}_\perp^2 g_2(\mathbf{k}_\perp^2, \epsilon, \Delta) &= \frac{1}{\epsilon^2} + \frac{1}{\epsilon} \ln \frac{\mu^2}{\Delta} + \frac{1}{2} \ln^2 \frac{\mu^2}{\Delta} + \frac{\pi^2}{3} + \mathcal{O}(\epsilon).
\end{aligned} \tag{A.7}$$

Therefore g_1 and g_2 can be expressed as

$$\begin{aligned} g_1(\mathbf{k}_\perp^2, \epsilon, \Delta) &= \left(\frac{1}{\epsilon} + \ln \frac{\mu^2}{\Delta} \right) \delta(\mathbf{k}_\perp^2) + \left[g_1(\mathbf{k}_\perp^2, \epsilon, 0) \right]_\infty \\ g_2(\mathbf{k}_\perp^2, \epsilon, \Delta) &= \left(\frac{1}{\epsilon^2} + \frac{1}{\epsilon} \ln \frac{\mu^2}{\Delta} + \frac{1}{2} \ln^2 \frac{\mu^2}{\Delta} + \frac{\pi^2}{3} \right) \delta(\mathbf{k}_\perp^2) + \left[\frac{1}{\mathbf{k}_\perp^2} \right]_\infty \ln \frac{\mu^2}{\Delta} + \left[\frac{\ln \mathbf{k}_\perp^2 / \mu^2}{\mathbf{k}_\perp^2} \right]_\infty. \end{aligned} \quad (\text{A.8})$$

Note that Eq. (A.8) is not strictly correct since the UV divergence is not peaked near $\mathbf{k}_\perp^2 \sim 0$. Therefore they are meaningful only inside the integral over the transverse momentum which reaches infinity. Then the UV divergence is correctly accounted for.

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